Lesson #2 Solving One and Two Variable Equations (1)

(Basic + Complex Equations)

Equation: is a statement that uses an equal sign to show that two quanitities or Expressions are equal.

Solution of an equation: is the value or values of the variable or variables that makes the equation true. (To find these values you must get a particular variable on one side of the equation by itself, set equal to another.

Example of Equations: $x+2=10$ $\frac{1}{2}x=6$

give examples After def. of Equation. $y-4=9$

Properties Used to Help Solve Equations: $2x=4$

Addition Property of Equality: You can add the same number to both sides of an Equation and the Statement will still be true. $a=b$ $a+2=b+2$ still equal

Subtraction Property of Equality: You can subtract the same number from both sides of an Equation and the statement will still be true. $a=b$ $a-2=b-2$.

Multiplication Property of Equality: Both sides of an equation can be multiplied by the same number, and the statement will be true.

$$a=b \quad a \cdot c=b \cdot c$$

Division Property of Equality: Division Property states both sides of an equation can be divided by the same number, and the Statement will be true

$$a=b \quad \frac{a}{c}=\frac{b}{c}$$

* With this in mind we can now start solving simple one-step equations.

$$x+2=10$$

goal: to get x by itself.

procedure: execute a series of steps/moves to → that don't change the problem. get the targeted variable to one side of the equal side and everything else to the otherside.

(Remember what you do to one side you MUST do to the other.) (Do this so you don't change Problem)

* but how do we get a term or constant from one side to the other? $x-2=10$

We exectue the Inverse Operation or opposite operation to get one thing from one side to another. (make sure though, what you do to one side, you do to the other.

$$\begin{array}{rcl} x-2 &=& 10 \\ +2 && +2 \end{array}$$

$$\boxed{x=12}$$

But is this correct?

Let's now check

$x=12$

original prob: $x-2=10$ plug in $x=12$ and check to see if you where right.

$12-2=10$

$10=10$ ✓ So therefore it is true and the solution or answer is correct

* Walk through process of each type.

* now that we understand the steps in solving these basic equations and how to check to make sure we are always correct, let's practice. Do this in the form of a pod challenge format.

(1) Addition Practice

1. $x-5=2$ $\boxed{7}$
2. $x-12=9$ $\boxed{21}$
3. $x-25=-18$ $\boxed{7}$
4. $x-3.7=-8.1$ $\boxed{-4.4}$
5. $a-4.1=6.3$ $\boxed{10.4}$
6. $22=-16+r$ $\boxed{-5}$
7. $-9=-6+x$ $\boxed{-3}$
8. $x-3=12$ $\boxed{15}$
9. $10=y-3$ $\boxed{13}$
10. $-5=x-7$ $\boxed{2}$
11. $x-6=4$ $\boxed{10}$
12. $x-5=17$ $\boxed{22}$
13. $-30=m-12$ $\boxed{-18}$
14.
15.
16.
17.

2 Subtraction Practice

1. $K+7=13$ $\boxed{6}$
2. $n+17=20$ $\boxed{3}$
3. $6+x=-1$ $\boxed{-7}$
4. $x+5=7$ $\boxed{2}$
5. $x+5=-8$ $\boxed{-13}$
6. $p+3=37$ $\boxed{34}$
7. $-14=y+8$ $\boxed{-22}$
8. $d+4\frac{1}{2}=3\frac{1}{6}$ $\boxed{-1\frac{1}{3}}$
9. $x+\frac{1}{2}=\frac{3}{4}$ $\boxed{\frac{1}{4}}$
10.
11.
12.
13.
14.
15.
16.
17.

3 Multiplication Practice

1. $\frac{x}{6}=8$ $\boxed{48}$
2. $-11=\frac{1}{4}w$ $\boxed{-44}$
3. $\frac{4}{7}x=\frac{9}{14}$ $\boxed{9/8}$
4. $\frac{x}{7}=8$ $\boxed{56}$
5. $\frac{3}{4}y=11$ $\boxed{\frac{44}{3}}$
6. $-20=\frac{1}{5}m$ $\boxed{-100}$
7. $8=-\frac{5}{12}n$ $\boxed{-96/5}$
8. $\frac{k}{9}=3$ $\boxed{27}$
9. $-\frac{5}{6}t=-15$ $\boxed{18}$
10. $\frac{3}{5}k=-21$ $\boxed{-35}$
11. $\frac{p}{4}=-6$ $\boxed{-24}$
12. $-\frac{7}{9}c=\frac{3}{5}$ $\boxed{-\frac{27}{35}}$
13. $\frac{2}{7}p=4$ $\boxed{14}$
14.
15.
16.
17.

4 Division Practice

1. $10z=-45$ $\boxed{-9/2}$
2. $-6x=14$ $\boxed{-7/3}$
3. $3r=-15$ $\boxed{-5}$
4. $6p=60$ $\boxed{10}$
5. $4x=68$ $\boxed{17}$
6. $5x=20$ $\boxed{4}$
7. $-8x=60$ $\boxed{-15/2}$
8. $12x=132$ 11
9. $-12=3m$ $\boxed{-4}$
10. $8y=24$ $\boxed{3}$
11. $-15=3x$ $\boxed{-5}$
12. $-p=-7$ $\boxed{7}$
13.
14.
15.
16.
17.

* now ask if there are any questions?

* Tell them that tomorrow we will be working on putting all of this together and combining a few of these into the same problem and creating Two step and multi-step problems. You

* leave them with some possible examples if have time

Two Step problems:

1. $5m+4=14$ $\boxed{2}$
2. $4x+5=17$ $\boxed{3}$
3. $8=-5m+6$ $\boxed{-2/5}$
4. $6r+12=42$ $\boxed{5}$
5. $32=-4q+8$ $\boxed{-6}$
6. $\frac{2}{5}x-\frac{1}{4}=\frac{3}{10}$ $\boxed{-11/8}$
7. $1/2n-\frac{1}{3}=\frac{3}{4}$ $\boxed{13/6}$
8. $-10=-2x+12$ $\boxed{11}$
9. $8w-4=28$ $\boxed{4}$
10. $9y+6=24$ $\boxed{2}$
11. $7m-5m=-12$ $\boxed{-6}$
12. $4r-9r=20$ $\boxed{-4}$

* Correct procedure when dealing with Two Step problems is to first do any addition or Subtration to get any constants or non targeted variable to one side and have the other side for just the targeted variable and any coefficient. Then you can do any multiplication or division to isolate the targeted variable by itself and get your soultion.

* Always remember to check your work by plugging it back in. You should Always know if you have the right answer or not.

Lesson # 2 Solving Multi-Step Equations + Solving Equations with Variables on Both Sides

* yesterday we finished up with solving two step equations such as below:

$$4x - 5 = 11$$
$$+5 \quad +5$$
$$\frac{4x}{4} = \frac{16}{4}$$
$$\boxed{x = 4}$$

* we also talked about how we check our work to make sure our answer is correct. (AKA: Plug N Chug)

Check: $x = 4$ $\quad 4x - 5 = 11$

$$4(4) - 5 = 11$$
$$16 - 5 = 11$$
$$11 = 11 \checkmark$$

So we know that our answer is correct.

* Today we are going to introduce multi-step problems which will simply ask you to simplify the expression on on side and create a Two-step equation then solve the simple two step equation like we did yesterday.

- Lets take for instance

$$5x + 8 - 3x + 2 = 20$$

* the steps in this are as follows:

(1) Simplify the expression to the left of the equals sign and forget about the right hand side.

ex: $5x + 8 - 3x + 2$ Simplify this

(combine like terms)

$$2x + 10$$

(2) Set the simplified expression equal to the Right and create a simple two-step equation.

ex: $2x + 10 = 20$

(3) Now solve the simple two-step problem like we have been doing:

ex $\quad 2x + 10 = 20 \quad\quad 2x = 10$

4. Now check your answer and make sure it works:

$x=5 \quad 5x+8-3x+2=20$

$5(5)+8-3(5)+2=20$

$25+8-15+2=20$

$33-15+2=20$

$18+2=20$

$20=20$ ✓ Yes, $x=5$ is correct.

* these are the steps that you should follow whenever you are trying to solve multi-step problems that are set equal to a constant.

* Now, practice a couple more like this one and have a couple students come to the board to solve them.

Other problems:

1. $3+x-10+4x=28$ $\boxed{x=7}$
2. $x+3(2x+4)=47$ $\boxed{x=5}$
3. $-2(x-2)+x=16$ $\boxed{x=-12}$
4. $5x-(x-3)-1=18$ $\boxed{x=4}$
5. $6x+2(7-x)+10=20$ $\boxed{x=-1}$
6. $3x+2-x+7=16$ $\boxed{x=3.5}$
7. $6(x-1)=36$ $\boxed{x=7}$
8. $5x-3(x-4)=22$ $\boxed{x=22}$
9. $\frac{3}{4}+\frac{1}{2}x+2=0$ $\boxed{x=-5\frac{1}{2}}$
10. $16x-4(3x-2)=40$ $x=$
11. $7x-8+5x+2=30$ $x=$

* Now, what happens if we have an Algebraic expression, like on the left side of these equations, set equal to another algebraic expression. How would we solve that?

Example: $4x+5=2x+11$

Well lets start off with a simple example that has a two-step equation set equal to a two step equation.

* for a problem like this, our goal is to solve for x and find out what x is equal to. So we will need to move all of the x's to one side and Isolate them by themselves, and move all other constants to the other. (we will use all of the operations we talked about yesterday to make these moves.)

So $4x + 5 = 2x + 11$

$$\begin{aligned} 4x + 5 &= 2x + 11 \\ -2x \quad &\quad -2x \\ 2x + 5 &= 11 \\ -5 \quad &\quad -5 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

<u>Now check</u> $x=3$

$4(3)+5 = 2(3)+11$

$12+5 = 6+11$

$17 = 17$ ✓ So, it is correct.

* Now what happens when it gets more complicated?

<u>Example</u>: $5(2x+4) - 2x = 6 + 2(3x+12)$

* What are the steps in solving this?

<u>Steps in Solving</u>

1. Forget about the expression on the right hand side of the equals sign, cover it up. Now simplify just the left hand side.

$5(2x+4) - 2x$

$10x + 20 - 2x = 8x + 20$

2. Forget about the expression on the left hand side of the equals sign, cover it up. Now simplify the right hand side.

$6 + 2(3x+12)$

$6 + 6x + 24 = 6x + 30$

3. Now that both sides are simplified, set them equal and bring all of the x's to one side and all the constants to the other.

$8x + 20 = 6x + 30$

$-6x \qquad -6x$

$$2x + 20 = 30$$
$$-20 \quad -20$$

$$2x = 10$$

4 Now you have just created a simple one-step equation. Solve the one-step equation for x.

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

5 Now Check your answer

$$x = 5$$

$$5(2(5)+4) - 2(5) = 6 + 2(3(5)+12)$$

$$5(14) - 10 = 6 + 2(15+12)$$

$$70 - 10 = 6 + 2(27)$$

$$70 - 10 = 54 + 6$$

$60 = 60$ ✓ Yes, it is correct.

* these are the steps that you should follow whenever you are trying to solve equations that have more complex expressions on either side of the equals sign.

* Now practice a couple more like this one and have a couple students come to the board to solve them.

other problems:
1 $8x = 3x + 20$ $\boxed{x=4}$
2 $3x + 4(2-2x) = 3(5x+1) - 15$ $\boxed{x=1}$
3 $6x = 4x - 10$

* now there are 2 cases that you might come across when trying to solve these problems.

example: $10 - 6x = -2(3x - 5)$

$$10 - \cancel{6x} = -\cancel{6x} + 10$$
$$+\cancel{6x} \quad +\cancel{6x}$$

$$10 = 10$$

when you come across a problem where the x's cancel out and you have a number equal to that it has an infinite number of answers. All possible solutions work. No mater what x you have you will get 10=10 which means it is true. (If you look closely at the simplified expressions on each side of the equals sign, you will realize that they are the <u>same</u>.) So All solutions or infinite # of solutions is the correct answer.

* Now you may come across another case that yields a very interesting answer like the following:

$$7x - 2 = 9x - 5 - 2x$$

$$\cancel{7x} - 2 = \cancel{7x} - 5$$
$$-7x \qquad -7x$$

$$-2 = -5$$

When you come across a problem when the x's cancel out on both sides and you are left with some number equal to some different number the the answer is that it is <u>never true</u>, and that there are <u>no solutions</u>, that would make these equals equal. They are complete opposites and will never be equal.

* Now that we have seen all of these types of problems it is time for you guys to practice them. In their pods they can work on a "pod Challenge" activity. Use some of the following problems

1. $6x = 3x + 27$ $\sqrt{x=9}$
2. $2+3(3x-6) = 5(x-3)+15$ $\boxed{x=4}$
3. $2(x+3) = 3(2x+2)-4x$ $\boxed{\text{All soultions}}$
4. $3(x+4) = 2(x+5)+x$ $\boxed{\text{No soultions}}$
5. $2x+4(x-3) = 2(3x+6)$ $\boxed{\text{All solutions}}$
6. $9x-5(2x+1) = 2(3x-6)-7x$ $\sqrt{\boxed{\text{No solutions}}}$

Lesson #2 Continued: Linear Equations is an equation of the form $y=mx+b$

Solving Equations with Variables on Both Sides & Solving Literal Equations (2 or More Variables)

* yesterday we finished up working on Solving equations with one Variable and variables on both sides like this

Ex #1 $5(2x+4)-2x = 6+2(3x+12)$

or

Ex #2 $6x = 4x-10$

Ex #1 $5(2x+4)-2x = 6+2(3x+12)$

$$10x+20-2x = 6+6x+24$$
$$8x+20 = 30+6x$$
$$-6x \qquad -6x$$
$$2x+20 = 30$$
$$-20 \quad -20$$
$$\frac{2x}{2} = \frac{10}{2} \qquad x=5$$

* Now we can check our answer to make sure it is right. Yesterday everyone was asking if it was right. You should Know if your answer is right by checking it, plugging in whatever answer you get into that particular variable and solve. You should end up getting, something equal to itself.

Like this

$20=20$ ✓ True, Correct answer.

(* when a number is equal to the same number it is always true and therefore the x you got is Correct.) Also Called the Identity of an equation

(* if you get some number equal to another number, it is never true and that answer you got would be wrong and not a solution to that problem.)

* Keep these two Statements in mind when we go over the next few problems.

* Now Check your answer from above

$x=5$ plug into problem

$$5(2(5)+4)-2(5) = 6+2(3(5)+12)$$
$$5(10+4)-10 = 6+2(15+12)$$
$$5(14)-10 = 6+2(27)$$

$$5(14) - 10 = 6 + 2(27)$$
$$70 - 10 = 6 + 54$$
$$60 = 60 \checkmark$$

So $x = 5$ is the right Answer

* now solve this equation

$$10 - 6x = -2(3x - 5)$$
$$10 - \cancel{6}x = -\cancel{6}x + 10$$
$$+\cancel{6}x \qquad +\cancel{6}x$$
$$10 = 10$$

* What does this mean? Remember when a number equals a number then it is always true and that this equation has infinitely many Solutions. (If you look really closely they are the exact same equations, so no matter what you plug in for x, the two side will always be equal. Infiniately many solutions.

* Now solve this equation

$$7x - 2 = \underline{9x} - 5 - \underline{2x}$$
$$7\cancel{x} - 2 = 7\cancel{x} - 5$$
$$-\cancel{7}x \qquad -\cancel{7}x$$
$$-2 = -5$$

* What does this mean? Remember when a number is set equal to another number then it is never true and could never be true. Because it can never be True there are <u>no solutions</u> to this particular equation. They will never be equal to each other.

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* Now we are going to build up to equations with 2 variables and solve for particular targeted variables. The procedures and steps will be very similar to those we used to solve one variable equations.

This is what we call a Literal <u>Equation</u>

<u>For example</u> ① $2x + 6y = 12$

① Solve for x:

② Solve for y:

Take for example : $2x + 6y = 12$

* The first thing that must be understood is what you are solving for.

Solve for x: Once you have established what you are solving for you will treat the other variable term as if it where a constant term and move it to the otherside.

So... Solve for x

$$\begin{aligned} 2x + 6y &= 12 \\ -6y &\quad -6y \end{aligned}$$

(leave the x variable and coefficient alone and move everything else away from the variable you are trying to solve for. Must do opposite to move terms.)

$$\frac{2x}{2} = \frac{12}{2} - \frac{6y}{2}$$

$$\boxed{x = 6 - 3y}$$

(because your dealing with more than 1 variable, your answer when finished will have the other variable in it.)

Now solve the same equation, but this time solve for y.

$$\begin{aligned} 2x + 6y &= 12 \\ -2x &\quad -2x \end{aligned}$$

(Leave y term alone and move x term out of there.)

$$\frac{6y}{6} = \frac{-2x}{6} + \frac{12}{6}$$

$$\boxed{y = -\frac{1}{3}x + 2}$$

* This process will even work for more Complicated two variable Equation. (Just remember Simplify both sides of equal sign first, then get all of the variables you are trying to solve for on one side and move everything else away from it. Treat the other variable as if it were another constant term.)

Ex. $-8x - 4y = 16$ Solve for y

$$\begin{aligned} -8x - 4y &= 16 \\ +8x \quad & \quad +8x \\ -4y &= 8x + 16 \\ \frac{-4y}{-4} &= \frac{8x}{-4} + \frac{16}{-4} \\ &\boxed{y = -2x - 4} \end{aligned}$$

Solve for x

$$\begin{aligned} -8x - 4y &= 16 \\ +4y \quad & \quad +4y \\ -8x &= 4y + 16 \\ \frac{-8x}{-8} &= \frac{4y}{-8} + \frac{16}{-8} \\ &\boxed{x = -\frac{1}{2}y - 2} \end{aligned}$$

* Now lets make it even more difficult. (No matter what you will still follow the same steps in solving.)

Ex. $-4x + 3y - 6 + 7x - 11y + 14 = 32$ Solve for y.

* First thing you need to do is simplify the left hand side of the equation, by collecting like terms.

$$\boxed{-4x}\ +3y\ -6\ \boxed{+7x}\ -11y\ +14$$

$3x - 8y + 8 = 32$ Now it's just like our previous problems

$$3x - 8y + 8 = 32$$
$$\quad\quad -8 \quad -8$$

(we are solving for y so move everything not Directly Attached to the y to the opposite side of the equal sign and <u>leave the y alone.</u>)

$$3x - 8y = 24$$
$$-3x \quad\quad -3x$$

$$\frac{-8y}{-8} = \frac{-3x}{-8} + \frac{24}{-8}$$

$$\boxed{y = \frac{3}{8}x - 3}$$

* Once you have moved everything not directly attached to the ~~to~~ the opposite side, then Split up the coefficient and variable by doing the opposite operation, <u>DIVIDING</u>.)

* Lets now take it one Step further and put in parenthesis.

$$3(x+5) + 4(2y+3) = 45 \quad \text{Solve for } x.$$

(again we treat it no different than any other Problem, just worry about simplifying the left side of the equation to make it easier and look something like this: $2x + 3y = 6$, Which we already know how to do.)

$$3(x+5) + 4(2y+3) = 45$$

(First Distribute and collect like terms to Simprfy the left Side.)

$$3x + 15 + 8y + 12 = 45$$

$$3x + 8y + 27 = 45$$
$$\quad\quad -27 \quad -27$$

$$3x + 8y = 18$$
$$\quad -8y \quad -8y$$

Now that its in a form like we have seen before Simply Solve like before.

$$\frac{3x}{3} = \frac{18}{3} - \frac{8y}{3}$$

$$\boxed{x = 6 - \frac{8}{3}y}$$

*It can Still get a little more complex so lets look at an example with Variable on both Sides of the equal sign.

Ex.

$$4(x+2)+3y = 3(y-2)+2x$$ Solve for x.

(Don't Panic Solve it the Same as ever other. Worry first about Distributing Collecting like terms and Simplifying each Side separetly.)

$$4(x+2)+3y = 3(y-2)+2x$$

$$4x+8+3y = 3y-6+2x$$

(Now that each side is simplified, get all the x's to one side, because that is what you are solving for. Then move every thing else not attached to the x Variable to the other side of the equal Sign.

$$4x+8+3y = 3y-6+2x$$
$$-2x \qquad\qquad -2x$$

$$2x+8+\cancel{3y} = \cancel{3y}-6$$
$$-\cancel{3y} \quad -\cancel{3y}$$

$$2x+8=-6$$
$$-8 \quad -8$$

$$\frac{2x}{2}=\frac{-14}{2}$$

$$\boxed{x=-7}$$

Now you should have no problem solving any Two Variable equation I give you.