

Lesson #2

(1)

Lesson #2 : Understanding and Simplifying Variables and Algebraic Expressions using the Product Properties of Exponents.

(Understanding Variables and Expressions and their Structure and terms.)

- Lets take this following example for instance:

$$4 + x$$

- In order to understand an expressions structure we need to understand two parts:

The Variable

* Variable can take on many different values, but at that particular time, its value is unknown.

- which is a symbol; usually a letter, that is used to represent an unknown number.

(in the case of our above Example x is our Variable.)

is a quantity whose value does not change.
(in the case of our above Example the 4 is our Constant and there is an imaginary 1 that is in front of the Variable x as well.)

* So

$4 + x$
↓
Constant
(the whole thing
is called an
Expression)

The constant:

* A constant has one single value the whole time and never changes.

Examples:

Identify the variables &
constants in
each expression

① $6 - 3x$

Variables: x

Constants: 6, 3

② $4 + 7x$

Variables: x

Constants: 4, 7

③ $7wz + 2qy$

Variables: w, z, y

Constants: 7, 2

* these two parts make up what we call an expression like the above (Ex. $4 + x$).

Expression: an expression is like a phrase of mathematical language that doesn't make sense

- (It is just a string of variables and numbers connected together by mathematical operations.)

- (An Expression cannot give you a solution.)

You can only Simplify an expression.)

④ $24wz + 12qy$

Variables: w, z, q Constants: 24, 12

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* When two or more quantities are multiplied together, each quantity is a factor of the product.

$$\begin{array}{c} 4 \times y \\ \downarrow \quad \downarrow \\ \text{factors} \end{array}$$

* The numeric factor of a product (4 in our case) is also called the numeric coefficient, or simply the Coefficient. (Implied coefficient - (xy) - in case like this there is an implied coefficient of 1 in front of the xy.)

Examples : Identify the factors and coefficients in each expression.

- (a) $7vw$ factors: 7, v, w - Coefficient: 7 Constant: 7 Variable: vw
- (b) $-5rst^4$ factors: -5, r, s, t Coefficient: -5 Constant: -5 Variable: r, s, t
- (c) $\frac{y}{3} \Rightarrow \frac{1}{3}y$ factors: $\frac{1}{3}, y$ Coefficient: $\frac{1}{3}$ Constant: $\frac{1}{3}$ Variable: y
- (d) $cd \Rightarrow 1cd$ factors: 1, c, d Coefficient: 1 Constant: 1 Implied coefficient Variable: c, d
- (e) $6de$ factors: 6, d, e Coefficient: 6 Constant: 6 Variable: d, e
- (f) $-2qrs^4$ factors: -2, q, r, s Coefficients: -2 Constant: -2 Variable: q, r, s
- (g) $\frac{y^8}{4} \Rightarrow \frac{1}{4}y^8$ factors: $\frac{1}{4}, y^8$ Coefficient: $\frac{1}{4}$ Constant: $\frac{1}{4}$ Variable: y
- (h) $b^8c^8 \Rightarrow 1b^8c^8$ factors: 1, b, c Coefficient: 1 Constant: 1 Implied coefficient Variable: b, c

* When individual expressions like we have been talking about are combined and separated by operation signs (+, -, like below), we are talking about multiple terms of an expression. A term that is in parentheses such as $(y+2)$ can

include a + or - and still be considered one term. $x + 4xym - \frac{6p}{(y+q)} - 8$

1 st	2 nd	3 rd	4 th
Term	Term	Term	Term

(all individual expressions combined to form a larger expression.)

$x + (y+3) + 6z - 2$

1 st	2 nd	3 rd	4 th
Term	Term	Term	Term

(Each individual term will be designated a particular term depending on the order in which they are in the sequence.)

Examples: Identify the terms in each expression

(a) $6xy + 57w - \frac{24x}{5y}$

$\underbrace{}_{1^{\text{st}}}$ $\underbrace{}_{2^{\text{nd}}}$ $\underbrace{}_{3^{\text{rd}}}$

(a₂)

$6x + (y+3) - 32z$

$\underbrace{}_{1^{\text{st}}}$ $\underbrace{}_{2^{\text{nd}}}$ $\underbrace{}_{3^{\text{rd}}}$

(b) $m + 3mn - \frac{5t}{(d+8)} - 9$

$\underbrace{}_{1^{\text{st}}}$ $\underbrace{}_{2^{\text{nd}}}$ $\underbrace{}_{3^{\text{rd}}}$ $\underbrace{}_{4^{\text{th}}}$

(c) $5ax + 42v - \frac{10a}{3x}$

$\underbrace{}_{1^{\text{st}}}$ $\underbrace{}_{2^{\text{nd}}}$ $\underbrace{}_{3^{\text{rd}}}$

(d) $x + 3xy - \frac{8d}{(j+4)} - 9$

$\underbrace{}_{1^{\text{st}}}$ $\underbrace{}_{2^{\text{nd}}}$ $\underbrace{}_{3^{\text{rd}}}$ $\underbrace{}_{4^{\text{th}}}$

- (e) The local telephone company uses the expression below to determine the monthly charges for individual customers

$$0.1m + 4.95$$

- # of terms? 2
- Identify constants? 0.1, 4.95
- Identify variables? m
- Identify coefficients? 0.1 (Not 4.95)

- (f) A car rental company uses the formula below to determine the charge to rent a car.

$$29.95 + 19.95d$$

- # of terms? 2
- Identify constants? 29.95, 19.95
- Identify variables? d
- Identify coefficients? 19.95 (Not 29.95)

Simplifying Expressions using the Product Property of Exponents

* As we all know, an exponent can be used to show repeated multiplication, take for instance

$$\underline{5^3}$$

- in this expression the base of a power (or base) (or number used as a factor) is 5.

- the exponent or power (in our case 3), indicates how many times the base is to be used as a factor (or how many times the base is multiplied together)
- So in the above example by normal exponential ~~Exponentiation~~ Simplification

Simplification (Simplify) - Means to perform all indicated operation to the furthest extent possible.

$$5^3 = 5 \cdot 5 \cdot 5 = \underline{125}$$

Simplifying.

Examples: More examples of this use of the ~~Product~~ ~~Property~~

Quotient to a power Rule:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

(a) $7^3 = 7 \cdot 7 \cdot 7 = 343$

(b) $(0.3)^4 = (0.3)(0.3)(0.3)(0.3) = .0081$

(c) $6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1296$

(d) $(0.8)^3 = (0.8)(0.8)(0.8) = 0.512$

(e) $\left(\frac{1}{3}\right)^6 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{729} \text{ or } \frac{1^6}{3^6} = \frac{1}{729}$

(f) $10^5 = 100,000$

(g) $\left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{32} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{32}$

(h) $10^3 = 1000$

* now with all of this in mind what can we do if we are presented with this following task

$$5^4 \cdot 5^5$$

- One way to do it would be like we would normally do with individual expressions with exponents and then multiply the final answers for both expressions together like this

$$5^4 \cdot 5^5 = (5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5 \cdot 5) =$$

$$5 \cdot 5 = 5^9 = 1,953,125$$

or realize that

$$5^4 \cdot 5^5 = 5^9 = 5^{4+5}$$

- So the product of powers whose bases are the same can be solved by evaluating the expression that results from the sum of the powers.
- This leads us to the Product of Property of Exponents rule, which states

IF m and n are real numbers and $x \neq 0$ then

$$x^m \cdot x^n = x^{m+n}$$
 (Generic Rule for numbers

(This doesn't work for
addition of properties
of exponents)

raise to other numbers as
long as you have equal
bases or like variables
with different powers.)

Examples: Using Product property of Exponents

a) $x^5 \cdot x^7 \cdot x^2$ (same base)

$$x^{5+7+2} = x^{14}$$

b) $m^3 \cdot m^2 \cdot m^4 \cdot n^6 \cdot n^7$ → can only combine
 $m^{3+2+4} \cdot n^{6+7}$ ones with like bases.

$m^9 n^{13}$ → we can't simplify or combine any more
cause bases are different

c) $b^5 \cdot b^4 \cdot b^2$

$$b^{5+4+2} = b^{11}$$

$$\textcircled{a} \quad \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{7} \cdot w^2 \cdot w^4 = \sqrt{3+5+7} w^{2+4} = \sqrt{15} w^6$$

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* remember an expression is fully simplified if it no longer contains exponents or if there are no factors with the same base.

* There is another rule that has developed from exponents and Product Property of Exponents that can help you get an estimate of expressions involving large products. Such as

$$1,127,000 \times 108 = ?$$

This rule is called Order of Magnitude, and is defined as converting both numbers to their nearest power of 10 and then multiplying those.

$$\text{So } 10^6 \cdot 10^2 = 10^8 = 100,000,000$$

This will give an estimate of the answer

$$1,127,000 \times 108 = 121,716,000$$

Real Life Examples : (go over in class)

@ In 2006, the fastest supercomputer's performance topped out at about one PFLOPS. One PFLOPS is equal to 10^3 TFLOPS. Each TFLOPS is equal to 10^{12} FLOPS. What was the computer's speed in FLOPS?

$$10^3 \cdot 10^{12} = 10^{15} = ? 1,000,000,000,000,000$$

(b) There are 10^6 exabytes in a yottabyte. There are 10^9 gigabytes in an exabyte. How many gigabytes are in a yottabyte?

$$10^6 \cdot 10^9 = 10^{15}$$